Design and Benchmarking of a Robust Strain-Based 3D Shape Sensing System

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Shape sensing can provide insight into the structural health and operating conditions of slender engineered surfaces in aerospace and maritime applications. Shape sensing often uses digital image correlation, inertial measurement, or inverse finite element methods, which can be impractical in applications involving real-time reconstructions, static and dynamic deformations, or dynamic mode shape identification, especially outside of controlled laboratory environments. This work describes ongoing efforts to design, build, and validate a low-cost and robust tool for real-time shape sensing. The sensor consists of a simple aluminum spar, instrumented with strategically placed strain gauges. A kinematic model is used to reconstruct bi-axial bending and torsional displacements along the spar. The model is validated against FEM simulations and canonical analytical solutions. A prototype of the spar is benchmarked using a motion capture system. The pre-calibrated errors are correlated with the direction of bending, permitting a directionally compensative calibration scheme. After calibration of the spar and kinematic model, validation errors are 2.18% in bending magnitude and .97° in bending direction. This work addresses the emerging need for new low-cost sensors for structural health monitoring in environments where other sensing methods do not perform well.

1. Introduction

Shape sensing refers to methods for inferring the deflections of flexible bodies. These deflections give important insight into the structural health and operating conditions of slender engineered lifting surfaces in aerospace and maritime application, where fluid loads are high and structural responses are impactful upon both the safety and the efficiency of those systems’ operation. Shape sensors have been applied to a variety of measurement tasks, including in-situ loaded deflection monitoring and normal mode shape reconstruction by Harwood et al. (2019a, 2019b), and the inverse modeling of loading conditions by Ward et al. (2018) and Young et al. (2018).

Fluid structure interaction (FSI) describes the coupling of a structure’s motions with the fluid dynamic forces on that structure. FSI in dense viscous fluids, known as hydroelasticity, is filled with rich physics that affect marine propulsion systems, biological flows, and energy harvesting, as well as other disciplines. Relative to FSI in lighter fluids and gases, hydroelastic interactions are highly nonlinear, and the frequency-domain approaches devised for aeroelastic systems are either extremely unconservative or completely unable to predict instabilities (Abramson and Ransleben, 1965). Robust adaptive control offers a path toward improving the safety and efficiency in such hydroelastic systems; however control strategies necessitate accurate models, which in turn requires suitable sensing to ensure observability of the system to be controlled. Moreover, an in-depth understanding of FSI physics enables designers to better cope with hydroelasticity, or to harness FSI responses for improved performance of compliant marine systems.

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Numerical predictions look to couple computational fluid dynamics (CFD) and finite element models (FEM) to capture complex hydroelastic interactions. These models require accurate resolution of energy transport, transfer, storage, and dissipation mechanisms - physical insights that are currently lacking. Experimental measurements, which seek to characterize the effects of fluid flows on the dynamics of a structure, are sparse and exhibit high uncertainties. This is due in part to the non-linear nature of fluid forces, and in part to the lack of established experimental methods – both for the acquisition and interpretation of data. Compounding the difficulty of experiments, accurate measurements are often difficult to obtain because of undesirable flow conditions, such as multiphase, corrosive, or opaque flows – conditions that are especially pervasive outside of well-controlled lab environments. Shape sensing can produce detailed experimental records of structural motions in-situ, improving the ability of researchers to investigate FSI both inside and outside of the lab. Shape sensing is usually achieved using digital image correlation, inertial measurements, or strain-based methods, which are briefly reviewed in the following paragraphs. A more detailed review of the state-of-the-art shape sensing technology can be found in (Di Napoli et al., 2019).

Inertial methods, accelerometry being most common, capture the dynamic motions of a structure and, through time-integration or the assumption of harmonic motion, produce estimated velocities and positions (Ortel et al., 2013). The most common application is experimental modal analysis, where the band-limited nature of inertial measurements is easily accounted-for. This method provides accurate results for dynamic systems, however accelerometers and gyroscopes are second-order sensors, which limits their useful frequency range. Inertial reconstruction also suffers from significant drift in the integrated kinematic quantities, especially when motions are not harmonic.

Digital image correlation (DIC) is a photogrammetric method, wherein high-resolution imaging is used to estimate the deflections of a structure relative to its undeformed condition (Savio, 2015). Randomized, high contrast “speckle” patterns visible to several cameras are mapped into deformation fields by cross-correlation. This method also produces accurate results, but it is reliant on the surrounding environment. A steady field of view with uninterrupted optical access is imperative to the success of optical reconstruction. Also, external mounting of cameras introduces concerns of corrosion and fouling due to the environment, making photogrammetry most effective in laboratory settings.

Another approach to shape sensing uses discrete measurements of strain on an object, which are processed using one of several modeling approaches to produce structural deformations. The inverse finite element method (iFEM) is one such model used to reconstruct a displacement field (Kefal and Oterkas, 2017), often with startling fidelity. The iFEM process, however, first requires a suitable finite element model of the structure in question. It follows that a new model and mesh, including detailed structural and material properties, must be generated and validated for each structure to be tested. Also, accurate solutions are dependent on the mesh size which implies that more computation time is needed for accurate reconstruction. These considerations suggest that iFEM, while invaluable for the measurement of complex, well-characterized structures, is not ideal for real-time reconstruction of deflections on objects that do not have pre-existing FE models.

Kinematic modeling is a lower-cost alternative to iFEM. A kinematic model uses strain-deflection relationships to reconstruct the shape of a deflected object without invoking any of the material properties of the structure. This makes kinematic models useful in cases where the details of the measured structure are unknown. Moreover, kinematic models are typically orders of magnitude less computationally expensive than iFEM, which makes them suitable for the real-time reconstruction of deflections. Di Napoli et al. (2019) used one such kinematic model, involving curve-fitting and integration of the strain distributions along a pair of one-dimensional beams. By reconstructing the deflections along the length the two non-collinear beams, the bending and twisting deflections of a

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flexible hydrofoil were measured with accuracies on par with camera-based methods. However, the kinematic model was limited to small relative deformations (10% of the span) beyond which the measurement error increased substantially. Moreover, the spar was limited to one-dimensional bending, so the system was unable to measure lead-lag bending deformations, and two independently-calibrated spars were needed to measure torsion.

The work presented here describes ongoing efforts to design, build, and validate a low-cost and robust tool for accurate shape sensing in real-time. The tool consists of a spar with a simple geometry, which is instrumented with strategically placed strain gauges. A kinematic model, similar to the one used by Di Napoli et al. (2019), is used to infer bi-axial bending and torsional displacements with large amplitudes along the spar. One or more spars may be affixed to or inserted into a parent structure with a more complex geometry, enabling measurement of the 3D deformations of the parent structure.

1.1. Objectives and Overview

The overall aim of this work is to develop a robust, easily-deployed, and low-cost shape sensor that improves upon the state-of-the-art solutions for structural health monitoring and on-line measurement of compliant lifting surfaces. To this end, this paper will address the following three sub-objectives: a) Develop a robust and computationally inexpensive kinematic model for both bending and torsion; b) validate the kinematic model using finite element method (FEM) simulations; and c) construct and benchmark a physical prototype of a shape-sensing spar undergoing static deformations in flexure (torsion to be address in future work).

This work will describe the design of the physical spar and show the development of an enhanced kinematic reconstruction algorithm, which is validated using synthetic data from finite element simulations. FEM and the kinematic model are also used to inform the placement of strain gauges on the spar. The physical prototype of the spar will be described and benchmarked against static reference measurements from a commercially-available motion-capture system. This work will contribute to the emerging need for new low-cost sensors for measuring fluid structure interactions in harsh or extreme environments where other sensing methods may not perform well.

2. Kinematic Model and Physical Spar Design

2.1 Spar Design

The physical spar is a circular rod with a length of one meter and a nominal diameter (2𝑟𝑐) of 3/8 in (9.5 mm). 6061 aluminum alloy was selected because it is lightweight, easily machined, and the resulting rod is very compliant. A circular cross section benefits from a simple relationship between shear strain and torsion angle but makes it difficult to ensure that the torsion of the spar tracks that of the parent structure. This issue is addressed by the addition of keyways to the spar, which were machined along the entire length of the rod. The implementation of keyways alters the relationship between the shear strain and the torsion angle. However, this is countered by the implementation of a linear shape factor to the shear-displacement equation.

Figure 1 shows the modeled cross-section of the spar and the angular locations of the strain gauges. Note that the while all the strain gauges are shown in figure 1, they are not necessarily placed at the same span-wise locations. In each of the two bending directions, the respective strain gauges (Omega SGD-3/350-LY13) are affixed to opposite sides of the spar and wired in half-bridge configurations. The two bending pairs are separated by a 90-degree angle. Shear strains are measured using Omega SGT-
2DD/350-SY13 gauges affixed 90 degrees from the keyway. The shear strain gauges include four grids on a single carrier, wired in a full-bridge configuration. Strain gauges were bonded to the spar with cyanoacrylate glue. The quantities and locations of each type of gauge are described in section 2.4.

![Diagram showing spar cross section and gauge locations](image)

*Figure 1 – Spar cross section and gauge locations. The blue and red gauges represent x-y bending gauges and the green represents a shear gauge.*

2.2 Kinematic Model

The kinematic model allows deflections to be reconstructed from measurements of strain only, without the need to know the detailed geometry or material properties of the specimen. Three coordinate systems are defined: a global Cartesian coordinate system (XYZ), a spar-aligned local Cartesian coordinate system (xyz), and a bending-plane coordinate system (SNT). Coordinate system XYZ is defined with its origin at the root of the spar. The X-axis points toward a keyway, the Z-axis points along the length of the undeformed spar, and the Y-axis follows the right-hand convention. The xyz coordinates are defined with x and y passing through perpendicular bending gauges (see figure 1) and z is tangent to the elastic axis of the spar. Coordinate system SNT is a rotation of coordinate system xyz with N defined positive toward the direction of curvature. S is coincident with z and T is defined using the right-hand convention.

The input for the kinematic model is a vector of 2p bending strain measurements (p measurements taken from each of the two orthogonal half-bridge arrays), along with l shear strain measurements. Independent (p − 1)-degree polynomial interpolants are fitted to the measured bending strains in each of the two (x, y) bending directions. The spar is discretized into M segments, each with length ΔS and the polynomial interpolants are evaluated at the M + 1 node locations to yield vectors \( \varepsilon_x \) and \( \varepsilon_y \). At each segment node, the interpolated strains are transformed into polar coordinates, expressed in terms of the resultant bending strains \( \varepsilon_N \) and local curvature directions \( \alpha \).

\[
\varepsilon_{N,i} = \sqrt{\varepsilon_{x,i}^2 + \varepsilon_{y,i}^2}
\]  

(1)
$$\alpha_i = \arctan \left( \frac{\varepsilon_{y,i}}{\varepsilon_{x,i}} \right)$$  \hspace{1cm} (2)$$

Figure 2 – An example of two beam segments undergoing locally-in-plane bending. The resultant strain is determined for each segment, which defines the local bending plane, in which the segment’s radius of curvature is defined and used to establish the location of the next segment node.

Deformation reconstruction proceeds in two steps. The bending strain measurements are used to calculate the bending deformation for each segment, as well as the orientation of each respective local bending plane. The combined effects of spar torsion and changing $\alpha_i$ are then superimposed to calculate the reorientation of the global bending direction. Figure 2 shows the discretized progression of two segments with their respective SNT coordinate systems, used to calculate local in-plane bending where $\hat{e}$ represents a unit vector in the subscripted direction. At each node, the bending strain used to find the local radius of curvature ($1/k$) using linearized beam theory given by equations 3 and 4.

$$\varepsilon_{N,i} = r_c k_i$$  \hspace{1cm} (3)

$$k = \frac{\partial \beta}{\partial S}$$  \hspace{1cm} (4)

With sufficiently fine discretization, each segment can be approximated by an arc of length,

$$\Delta S = \frac{L}{M}$$  \hspace{1cm} (5)

where $L$ is the total spar length. The radius of curvature is used to find the local arc angle $\beta$ using,

$$\beta_i = \Delta S k_i.$$  \hspace{1cm} (6)
Figure 3 – Spar torsion for one segment with the S axis coming out of the page.

This local arc angle is then used to find the coordinates of the \( i + 1 \) point in the \( i^{th} \) SNT coordinate system using equations 7 and 8:

\[
\vec{S}_i = \frac{\sin \beta_i}{k_i}; \\
\vec{N}_i = \frac{(1-\cos \beta_i)}{k_i}.
\] (7) (8)

Using figure 2, the following rotation matrix may be used to express the orientation of the \( i^{th} \) SNT coordinate system in the \((i - 1)\) SNT coordinate system:

\[
\begin{bmatrix}
\hat{e}_{N_{i-1}} \\
1 \\
\hat{e}_{S_{i-1}}
\end{bmatrix} =
\begin{bmatrix}
\cos \beta_{i-1} & -\sin \beta_{i-1} & 0 \\
0 & 0 & 1 \\
\sin \beta_{i-1} & \cos \beta_{i-1} & 0
\end{bmatrix}
\begin{bmatrix}
\hat{e}_{N_{i}} \\
1 \\
\hat{e}_{S_{i}}
\end{bmatrix}.
\] (9)

Figure 3 shows a cross section of the spar with the \( xyz \) and \( SNT \) coordinate systems shown; axes \( S \) and \( z \) are pointing out of the page. The direction of principal strain, which defines the \( S-N \) planes in which bending is resolved for each segment, changes along the length of the spar as a result of both torsion of the spar and changes in the direction of the resultant bending strain. Torsion is accounted for using the shear strain gauges and using the linear relationship between the shear strain, \( \varepsilon_{s\theta} \), and the twist rate, \( \Delta \theta \), given in equation 10.

\[
\Delta \theta_i = \frac{\varepsilon_{s\theta_i}}{r_c} \Delta S
\] (10)
\( \theta \) represents the rigid body rotation of both the \( xyz \) and the \( SNT \) coordinate systems. A change in the direction of the principal bending strain is represented by \( d\alpha \), where \( \alpha \) represents the rotation of the \( T-N \) axes relative to the \( x-y \) axes. The local bending plane is then rotated by an angle taken to be the sum of the two components,

\[
\varphi_i = \theta_i + d\alpha_i \tag{11}
\]

Similar to bending, the \( i^{th} \) \( SNT \) coordinates are expressed in the \( (i - 1) \) \( SNT \) coordinate system using equation 12.

\[
\begin{bmatrix}
\hat{e}_{N_{i-1}} \\
\hat{e}_{T_{i-1}} \\
1
\end{bmatrix}
= 
\begin{bmatrix}
\cos\varphi_i & \sin\varphi_i & 0 \\
-sin\varphi_i & \cos\varphi_i & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{e}_{N_i} \\
\hat{e}_{T_i} \\
1
\end{bmatrix} \tag{12}
\]

Combining the torsion and the bending components, the \( i^{th} \) \( SNT \) coordinate system can be expressed in the \( (i - 1) \) \( SNT \) coordinate system using the following rotation matrix:

\[
R_i^{i-1} = 
\begin{bmatrix}
\cos\varphi_i & \sin\varphi_i & 0 \\
-sin\varphi_i & \cos\varphi_i & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos\beta_{i-1} & -\sin\beta_{i-1} & 0 \\
0 & 0 & 1
\end{bmatrix} \tag{13}
\]

By pre-multiplying all rotation matrices up to and including \( R_i^{i-1} \), the \( i^{th} \) node-fixed coordinate system can be expressed in the global Cartesian coordinate system. Thus, reconstruction is performed by marching along the length of the spar, transforming local coordinates into global coordinates, and using a forward Euler integration. The complete local-coordinate system rotation with forward Euler integration becomes,

\[
\begin{bmatrix}
X_i\hat{e}_X \\
Y_i\hat{e}_Y \\
Z_i\hat{e}_Z
\end{bmatrix}
= 
\begin{bmatrix}
\cos\gamma & \sin\gamma & 0 \\
-sin\gamma & \cos\gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\left[ \prod_{m=1}^{i} R_m^{i-1} \right]
\begin{bmatrix}
N_i\hat{e}_{N_i} \\
T_i\hat{e}_{T_i} \\
S_i\hat{e}_{S_i}
\end{bmatrix}
+ 
\sum_{n=1}^{i-1} \begin{bmatrix}
N_n\hat{e}_{N_n} \\
T_n\hat{e}_{T_n} \\
S_n\hat{e}_{S_n}
\end{bmatrix} \tag{14}
\]

Note that \( \gamma \) is included as a rigid-body rotation at the root, used to transform the coordinate system from \( SNT \) to \( XYZ \).

2.3 FEM Model

To assist in design decisions, and to perform a synthetic benchmarking of the kinematic model, FEM simulations of the spar were performed. All FEM simulations were performed in Calculix using a non-linear solver in order to support cases with large deformations (Dhonadt, 2017). The model was composed of 36,000 hexahedron-type elements as shown in figure 4. In all simulated cases, the spar was cantilevered, with nodes at the clamped end constrained in all degrees of freedom. Numerical probes were used as virtual strain gauges, values from which were provided as inputs to the kinematic model.
2.4 Sensor Placement

The kinematic model is sensitive to the number of sensors and sensor placement due to the least squares curve-fitting scheme. The effects of sensor placement and quantity were investigated using FEM by simulating a single case with static deformation in response to a point load applied at the free tip. Bending reconstruction was performed using 18 different quantities and placements of the probes. The bending strain gauge distribution schemes investigated were linear (selected for simplicity and ease of application), half cosine (selected to concentrate measurements near the static end of the cantilevered beam), and half Chebyshev root spacing (selected to compliment the polynomial fitting scheme by improving interpolant stability). The unitless Sobolev norm, shown in equation 15, was used to quantify the error between the FEM results and the reconstructed deformations. $V_R$ represents the reconstructed data and $V_S$ represents the simulated data. The resulting error ranges from zero to one where an error of zero means the data are perfectly similar and one means the data are completely independent.

$$\text{Error} = \frac{|V_R - V_S|}{|V_R| + |V_S|}$$

(15)

Figure 5 shows the results from the sensor placement investigation. A half-cosine spacing with four bending strain locations was selected, based both upon these results and upon the desire to minimize measurement channels. The number of physical channels needed would be $(2p) + l$; $2p$ channels for two normal bending directions and, $l$ channels for torsion measurement. A total of twelve channels were available, which restricts the number of gauges to eight bending strain gauges (four for each direction) and four shear strain gauges. Within this limitation, the half-cosine spacing offers the lowest error – at least for cases with simple loading. Torsion and shear strain are related by a first-order equation, so a piecewise linear interpolant was selected for shear strain and the shear strain gauges were uniformly distributed along the length of the spar. The final gauge locations are shown in figure 6.
3. FEM and Analytical Validation

The performance of the kinematic model, separate from the strain measurements themselves, was assessed using synthetic data from FEM simulations. As before, discrete strain measurements were sampled from the strain field data at locations corresponding to the physical strain gauges and used as inputs to the kinematic model. The resulting bending and twisting deflections were compared to the simulated results. A range of loading conditions were evaluated, including point forces, opposing forces, and distributed forces both with and without point or distributed twisting moments. Table 1 shows the results from the FEM validation for all loading conditions, all of which demonstrate good agreement. The largest dimensionless torsion error occurs for the case with distributed force and moment, however the dimensional error in this case was only about one tenth of a degree. The largest dimensionless bending error occurs for the case with opposing point forces; one at the middle of the spar and one at the free tip. The elevated error is a result of the least squares curve fit. The actual bending strain distribution had a higher order than the third order polynomial used as an interpolant. The interpolation error thus propagates into a higher bending error. Figure 7 shows an example of the good agreement between the reconstructed and simulated results for the case with combined bending and twisting concentrated at the free tip (case #3).
Table 1 – FEM validation results. Errors are expressed using the Sobolev norm.

<table>
<thead>
<tr>
<th>Case</th>
<th>Bending Error</th>
<th>Torsion Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1  Point Bending Load</td>
<td>.0024</td>
<td>-</td>
</tr>
<tr>
<td>#2  Distributed Bending Load</td>
<td>.0034</td>
<td>-</td>
</tr>
<tr>
<td>#3  Point Bend and Twist</td>
<td>.0029</td>
<td>.0061</td>
</tr>
<tr>
<td>#4  Distributed Bend and Twist</td>
<td>.0083</td>
<td>.129</td>
</tr>
<tr>
<td>#5  Opposing Bending Loads</td>
<td>.09</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 7 – Results for the FEM validation case including bending and twisting at the free tip (case #3 in table 1). Clockwise from upper left: illustration of the load on the spar model; bending deflection parallel to tip loading; bending deflection normal to tip loading; torsional deflection. Note that the results are shown in a global coordinate system defined by $\gamma = 45^\circ$ in equation 14.

The kinematic model was also validated using known analytical strain distributions for canonical deformations. Two analytical cases were selected: bending into a closed circle, given a constant strain equal to the spar radius divided by the desired radius of the circle; and the reconstruction of a helical spiral, given by a constant bending strain and a constant shear strain. Similar to the FEM validation, discrete strain values were taken from the analytical strain field and used as inputs for the reconstruction model. While the results are omitted here for the sake of brevity, the kinematic model produced the theoretical shapes exactly in both cases. Both synthetic and analytical benchmarking demonstrated that the kinematic model was able to reconstruct large deformations accurately.
4. Experimental Benchmarking

4.1 Experimental Setup

The physical shape sensing spar was benchmarked using the VICON Tracker 3 motion capture system (VICON, 2015). Steady state bending data were simultaneously captured from the strain gauges and the motion capture system. Four tracking locations were established along the spar’s length, shown in Figure 8. The fifth set of markers, closest to the root, was omitted because the relative deflections at that location were too small for the motion capture system to produce reliable data. At each location, a 3D printed block was affixed with reflective markers (visible in Figure 7a), which were used to construct a local plane (co-planar with the \( x, y \) and \( N, T \) planes defined in Section 2.2). The translations and rotations of each plane were reconstructed in post-processing, using the unloaded state as the undeformed condition. Point loads were applied to the tip of the spar. The loads were applied at directions (\( \alpha_{\text{tip}} \)) varying from 0 to 315° with respect to the global X-axis, which was taken to be aligned with a keyway as shown in Figure 7b. Note that this definition of a global coordinate system corresponds to a rigid body rotation of \( \gamma = 45^\circ \) in equation 14. A total of 80 trials were conducted, with a zero-load reference recorded immediately before each trial.
4.2 Pre-calibrated Results

Figure 9 shows the bending magnitudes for the first 40 test cases, with the bending directions ($\alpha_{tip}$) indicated by line color. All data are normalized by the displacement magnitude of the VICON measurement location (a) nearest the free tip, denoted $\delta_a$. This figure shows that there is significant variability in the strain gauge data, relative to the motion capture data. On the other hand, when normalized by the maximum deflection reconstructed from strain-gauge data, the variance in the motion capture data becomes large, while the strain gauge deformations collapse together to describe a common shape function. Based on the maturity of the system, we chose the motion capture data as the more reliable of the two, but a third measurement method should be included in future work.

Figure 10 shows the comparisons of the strain gauge reconstruction versus the VICON measurements at the four VICON measurement locations (a) – (c). For each bending direction, the deflection patterns for the VICON and reconstructed measurements are in qualitative agreement, however there is visible error in both the magnitude and the direction of the reconstructed measurements. In most cases, the strain gauge reconstruction under-predicts the deflection magnitude and the directional error tends to be biased clockwise. Both Figure 9 and Figure 10 imply that the bending errors, both in magnitude and direction, are correlated with the bending direction, $\alpha_{tip}$.
4.2 Spar Calibration

Figures 11 and 12 illustrate the effect of bending direction on the relative bending magnitude and direction error, respectively, at each measurement location. The patterns in the measurement errors suggest a correlation with the bending direction. The errors were fit using a three-term sine series for each, shown in equations 16 and 17:

\[
E_M(\alpha_{SG}) = \sum_{i=0}^{3} A_i \sin (B_i \alpha_{SG} + C_i);
\]

\[
E_D(\alpha_{SG}) = \sum_{i=0}^{3} a_i \sin (b_i \alpha_{SG} + c_i).
\]

Regressions produced \(R^2\) values of greater than .95, implying strong correlation between the bending magnitude errors and the bending direction.

The fitted errors in equations 16 and 17 were used to implement a directionally compensative calibration scheme. At each of the four VICON measurement locations, the bending direction and magnitude were estimated and a linear interpolation was used for points located between these locations. The percentage error correctors, \(E_M\) and \(E_D\) are then cast into a multiplicative gain which is applied to the raw data.
Figure 11 – Reconstructed deflection bending magnitude errors with least-squares curve fit at each of the VICON measurement locations; Top left - .984m, Top right - .810m, bottom left - .482, bottom right - .277m. The error at each location can be approximated using a three-term sine series regression.

Figure 12 – Reconstructed deflection bending direction errors in radians with least-squares curve fit at each of the VICON measurement locations; Top left - .984m, Top right - .810m,
Figure 13 – Bending magnitude for validation dataset cases normalized by the furthest VICON bending measurement ($\delta_F$). The variability in the strain gauge measurements is greatly reduced.

4.4 Calibration Benchmarking

The directionally compensative calibration was developed using a training data set of 40 different static loading conditions to fit error terms. A second independent validation data set of 40 loading conditions was used to assess the calibration error. Figure 13 shows the normalized deflections of both the VICON and the calibrated shape sensing spar results. The deflections still show some variability with direction, but it is much-reduced relative to figure 8; the calibrated data show variability similar to that of the VICON data. Figure 14 shows the complete results of the calibrated trials, again presented as polar plots at the four VICON measurement locations. The calibrated results, shown in green, exhibit much lower errors in bending (2.18%) and in direction (97°) for the bending range of zero to 220mm. The calibration regime is shown to correct both the magnitude error and the directional error to produce extremely accurate measurements of bending deflections along the length of the spar without any external reference data.

5. Discussion

The preliminary static test results show an average bending error of about 8.4% before any calibration. That bending magnitude error is highly correlated with the bending direction. At the spar’s free tip, the highest errors were observed at 90° from the keyway and the lowest errors were observed in the direction of the keyways; the exact opposite was observed near the spar’s root. These error were used to inform a directionally compensative calibration scheme which was evaluated using an independent dataset. The calibration was shown to correct both the magnitude and direction errors. More significantly, the corrective gains are calculated using the raw strain measurements, so no external reference is required.
Residual error may be indicative of nonlinearity introduced by the physical application of the strain gauges (e.g. adhesion or alignment). It should also be noted that the normalized VICON measurements do not all collapse in Figure 13. In measurement locations D and C, the strain gauge reconstruction method exhibits lower variability. Because locations D and C are located closer to the root, the relative change in marker position at each is much smaller than at other measurement locations. This implies that the precision of the motion capture system – traced to camera resolution, lens linearization, and camera extensive properties – may be insufficient to resolve small motions.

The shape sensing spar’s performance in torsion was also intended to be evaluated. However, the VICON resolution proved to be too course to reliably measure the rotations of individual section planes. For the same reasons described above, it is likely that the motion camera system simply does not possess the precision required to benchmark the spar’s performance in torsion. To remedy this, markers are being mounted further from the elastic axis of the spar in ongoing experiments. Nonetheless, future work should seek to benchmark both the bending and the torsional performance of the shape sensing spar using a separate method to ensure a high quality reference for both systems.

6. Conclusions and Future Work

Real-time shape sensing provides vital insight into the structural health and operating conditions of marine structures. Although accurate state-of-the-art methods exist, they are often best suited for laboratory settings. This paper outlines an approach to shape reconstruction using a kinematic model and conventional foil strain gauges intended to be used in both laboratory and field settings.
This work presents a low-cost and robust solution for the shape sensing of lifting surfaces in complex flows. A robust and computationally inexpensive kinematic model, which is independent from structural properties, ensures that the model may be applied to the real-time deflection reconstruction of a wide range of structures. The proposed kinematic model takes advantage of the kinematic relationship between strains and deflections by using a least-squares curve fit through discrete span-aligned bending strain gauges. A local coordinate system integration approach allows the kinematic model to reconstruct both small and large deflections. On a single-core processor, the kinematic model can run on the order of 100’s of reconstructions per second, making the model suitable for on-line reconstructions during experiments or operation. The model also uses shear strain measurements to reconstruct torsion along the spar, though torsional validation is left as a topic for future work at this point. The kinematic model was validated by using discrete strain measurements, sampled from FEM solutions and canonical analytical solutions, as inputs to reconstruct the deflections.

A prototype of the spar was benchmarked using an established motion capture system. Data show that the bending magnitude error has an average of 8.4% and a direction error of 4° before calibration. The bending magnitude and direction error was found to be highly correlated to the bending direction. These errors were included into a directionally compensative calibration. Post-calibration error were found to be an average of 2.18% for bending magnitude and 0.97° for bending direction.

7. Acknowledgements

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8. References


